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Weakly non-linear excitations in an antiferromagnetic film

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Abstract. Effective dynamic equations describing weakly non-linear exchange–magnetostatic excitations are derived for the antiferromagnetic film with an 'easy-plane' anisotropy. The existence of 'algebraic' solitons has been predicted. The conditions for the excitation of soliton states are investigated for various values of magnetic field perpendicular to the surface of the film and film thickness. The soliton relaxation is discussed.

1. Introduction

Interest in the non-linear properties of antiferromagnetic films has arisen quite recently. It is caused by two circumstances: the fundamental physical features of these materials, and possible technical applications [1, 2]. The peculiarities of propagation of the activation mode in an antiferromagnet were studied in [3]. In the present report the interaction of zero-gap exchange–magnetostatic spin waves in an antiferromagnetic film with an 'easy-plane' anisotropy is investigated.

The problem is significantly complicated when the magnetostatic interaction is taken into account even for waves propagating along one assigned direction. The problem becomes not only three dimensional but also non-local. We have deduced a set of coupled equations which describes weakly non-linear non-local dynamics of zero-gap spin waves and discussed the condition necessary for soliton existence in an antiferromagnetic thin film.

2. Problem formulation

Consider an antiferromagnetic thin film with an 'easy-plane' anisotropy. The constant magnetic field and anisotropy axis are directed along the z axis which is perpendicular to the surface of the film. It is convenient to use the following parametrization for the magnetization vectors of sublattices:

$$M_i = M_0\{\cos\theta_i \cos\varphi_i, \cos\theta_i \sin\varphi_i, \sin\theta_i\} \qquad i = 1, 2$$

where M_0 is the saturation magnetization. The energy density can be written in terms of independent variables θ_i and φ_i [4]:

$$w = w_1 + w_2$$

$$w_1 = M_0^2/2\{\alpha_1[(\partial_i\theta_1)^2 + (\partial_i\theta_2)^2 + \cos^2\theta_1(\partial_i\varphi_1)^2 + \cos^2\theta_2(\partial_i\varphi_2)^2] + 2\alpha_2[\partial_i\theta_1\,\partial_i\theta_2(\sin\theta_1\sin\theta_2\cos\phi + \cos\theta_1\cos\theta_2) + \partial_i\varphi_1\,\partial_i\varphi_2\cos\theta_1\cos\theta_2\cos\phi$$

$$- \partial_i\theta_1\partial_i\varphi_2 \sin\theta_1\cos\theta_2\sin\phi + \partial_i\theta_2\partial_1\varphi_1\cos\theta_1\sin\theta_2\sin\phi]$$
$$+2\delta(\cos\theta_1\cos\theta_2\cos\phi+\sin\theta_1\sin\theta_2)+\beta_1(\sin\theta_1-\sin\theta_2)^2-\beta_2(\sin\theta_1+\sin\theta_2)^2$$
$$- 2(\sin\theta_1+\sin\theta_2)h\} \quad (1)$$

$$w_2 = -M_0^2/2\{[\cos\theta_1\cos\varphi_1+\cos\theta_2\cos\varphi_2]h_x^{(m)} + [\cos\theta_1\sin\varphi_1+\cos\theta_2\sin\varphi_2]h_y^{(m)}$$
$$+ (\sin\theta_1+\sin\theta_2)h_z^{(m)}\} \quad (2)$$

where $\partial_i = \partial/x_i$, α_1, α_2 and δ are the exchange interaction constants (α_1, α_2 and δ describe non-uniform and uniform exchange interactions, respectively), β_1 and β_2 are the magnetic anisotropy constants ($\beta_1 > 0$ for an antiferromagnet with 'easy-plane' anisotropy), $\boldsymbol{H}$ is the external magnetic field, $h = H/M_0$, $h^{(m)} = H^{(m)}/M_0$, $\phi = \varphi_1 - \varphi_2$. Note that $\boldsymbol{H}^{(m)}$ is the field defined by the equations of magnetostatics:

$$- \Delta\varphi + 4\pi\,\mathrm{div}(\boldsymbol{M}_1+\boldsymbol{M}_2) = 0 \qquad \boldsymbol{H}^{(m)} = -\nabla\varphi \quad (3)$$

$$\Delta\tilde{\varphi} = 0 \qquad \tilde{\boldsymbol{H}}^{(m)} = -\nabla\tilde{\varphi} \quad (4)$$

where φ and $\tilde{\varphi}$ are magnetic scalar potentials inside and outside the slab, respectively.

The ground state of the system is determined by the energy minimum conditions $\int(\partial w/\partial\theta_i)\,\mathrm{d}\boldsymbol{r} = \int(\partial w/\partial\varphi_i)\,\mathrm{d}\boldsymbol{r} = 0$. As a result the equilibrium values of the angles are defined by

$$\varphi_1^0 - \varphi_2^0 = \pi \qquad \theta_1^0 = \theta_2^0 = \theta^0 \qquad \sin\theta^0 = h/2(\delta+4\pi-\beta_2). \quad (5)$$

In the case considered, the equations describing the dynamics of non-linear excitations can be written by the Lagrange function with the density

$$L = (M_0/g)(\sin\theta_1\partial_t\varphi_1 + \sin\theta_2\partial_t\varphi_2) - w \quad (6)$$

where $\partial_t = \partial/\partial t$ and g is the magnetomechanical ratio. The system of these equations takes the following form:

$$\begin{aligned}&\partial L/\partial\theta_i - \partial_j[\partial L/\partial(\partial_j\theta_i)] - \partial w_2/\partial\theta_i = 0\\ &\partial L/\partial\varphi_i - \partial_t[\partial L/\partial(\partial_t\varphi_i)] - \partial_j[\partial L/\partial(\partial_j\varphi_i)] - \partial w_2/\partial\varphi_i = 0 \qquad i = 1,2.\end{aligned} \quad (7)$$

The presence of additional terms $\partial w_2/\partial\theta_i$ and $\partial w_2/\partial\varphi_i$ in (7) means that the field $\boldsymbol{H}^{(m)}$ is not independent and defined by resulting magnetization $\boldsymbol{M} = \boldsymbol{M}_1 + \boldsymbol{M}_2$. Consequently, in varying the Lagrangian in fields θ_i and φ_i the variations in the field $\boldsymbol{H}^{(m)}$ should be taken *into account. Thereby the relation [4] is fulfilled:*

$$\delta\int_v \mathrm{d}\boldsymbol{r}\,\frac{\boldsymbol{H}^{(m)}\cdot\boldsymbol{M}}{2} = \int_v \mathrm{d}\boldsymbol{r}\,\boldsymbol{H}^{(m)}\cdot\delta\boldsymbol{M}$$

where the integration is performed over a crystal volume. The boundary conditions in the case of free spins at the surface of the slab are defined by the relations

$$\begin{aligned}&\partial_z\theta_i|_{0,d} = \partial_z\varphi_i|_{0,d} = 0 \qquad \varphi|_{0,d} = \tilde{\varphi}|_{0,d}\\ &- \partial_z\tilde{\varphi}|_{0,d} = [-\partial_z\varphi + 4\pi M_0(\sin\theta_1+\sin\theta_2)]|_{0,d}.\end{aligned} \quad (8)$$

Here d is the slab thickness.

3. Effective equations of motion and their solutions

The main task of our paper is to derive the effective evolution equations for weakly non-linear exchange–magnetostatic waves on the basis of the complete macroscopic equations (3), (4) and (7) with the boundary conditions (8). For definiteness, assume that the waves propagate along the x axis. The problem involves two characteristic space scales: the slab thickness d and the size Δ of the magnetic inhomogeneity (of the soliton). We shall deal with the weakly non-linear waves under the following conditions:

$$\Delta \gg \max\{(\alpha + \alpha'' \tan^2\theta^0)/2\pi d, d\} \tag{9}$$

$$d/\delta\Delta \ll [h/2(\delta + 4\pi - \beta_2)]^2 \ll 1. \tag{10}$$

Here $\alpha = (\alpha_1 + \alpha_2)/2$ and $\alpha'' = (\alpha_1 - \alpha_2)/2$. It can be shown that the spin-wave spectrum of an antiferromagnet with 'easy-plane' anisotropy has two branches: the activation branch and zero-gap (Goldstone) branch. We are interested only in the Goldstone exchange–magnetostatic mode. The dispersion relation in the given range of magnetic fields and space–time scales is

$$\omega^2 = (2gM_0)^2\alpha'' k_x^2 \cos^2\theta^0(\delta + 4\pi - \beta_2 - 2\pi d|k_x|). \tag{11}$$

Note that the demagnetizing fields which determine the magnetostatic part of the spin-wave spectrum can be estimated using the convenient method proposed in [5]. To derive the effective evolution equations for Goldstone modes, we shall use a version of the reductive perturbation theory based on coordinate stretching [6]. The forms of the scale transformations in the reductive perturbation theory are different inside and outside the film. We shall look for the solution of (3), (7) and (8) inside the slab in the form

$$\begin{gathered}
\varphi_i = \varphi_i^{(0)}(\xi, \tau, z) + \sum_{n=1}^{\infty} \varepsilon^n \varphi_i^{(n)}(\xi, \tau, z) \qquad \xi = \varepsilon(x + st) \qquad \tau = \varepsilon^2 t \\
\theta_i = \theta^0 + \sum_{n=1}^{\infty} \varepsilon^n \theta_i^{(n)}(\xi, \tau, z) \qquad \varphi = \varphi^{(0)}(\xi, \tau, z) + \sum_{n=1}^{\infty} \varphi^{(n)}(\xi, \tau, z)\varepsilon^n \\
s = 2M_0 g \cos\theta^0[\alpha''(\delta + 4\pi - \beta_2]^{1/2}.
\end{gathered} \tag{12}$$

Here ε is the small parameter characterizing the deviation of the system from the equilibrium state $d \cot\theta^0/\delta\Delta \sim \theta_i - \theta^0 \sim O(\varepsilon) \ll 1$. The scale transformations are introduced to match the space–time response of the system to dispersion law (11) and to obtain a balance between the dispersion and quadratic non-linearity. Outside the film we look for the solution of the magnetostatic equation (4) in the form

$$\tilde{\varphi} = \tilde{\varphi}(\eta, z, \tau) + \sum_{n=1}^{\infty} \varepsilon^n \tilde{\varphi}^{(n)}(\eta, z, \tau) \qquad \eta = x + st \qquad \tau = \varepsilon^2 t. \tag{13}$$

The effective equations for the Goldstone excitations are obtained as a result of the combination of two versions of the perturbation theory at the boundary of the film. A similar approach was used earlier in order to describe the propagation of the waves in a stratified fluid [7]. Note that each order of the perturbation theory involves terms depending on the z coordinate. The successful application of the method above is based on the opportunity of separating the variables ξ and z (or η and z) in the equations of the perturbation theory. A set of equations of perturbation theory is solved sequentially, beginning with the lowest order of ε. The corresponding boundary conditions can be obtained from the expansion of the conditions (8) in a power series of ε. It is important that the conditions

$$\partial_z \theta_i^{(n)}|_{0,d} = \partial_z \varphi_i^{(n)}|_{0,d} = 0 \qquad (i = 1, 2)$$

give rise in the lowest order of ε to the z-independent functions: $\varphi_i^{(0)}$, $\varphi_i^{(1)}$, $\theta_i^{(1)}$ $(i = 1, 2)$; $\varphi_1^{(n)} + \varphi_2^{(n)}$, $(n = 2, 3)$; $\theta_1^{(2)} + \theta_2^{(2)}$. This simplifies the problem considerably; however, it is not of fundamental importance as the method used is applicable to more complicated boundary conditions. The zero and first order in the parameter ε perturbation theory equations have the following solutions:

$$\begin{gathered}
\varphi_1^{(0)}(\xi, \tau) = \pi + \varphi_2^{(0)}(\xi, \tau) \qquad \theta_1^{(1)} = \theta_2^{(1)} = \gamma \partial_\xi \varphi_1^{(0)} \qquad \gamma = [\alpha''/(\delta + 4\pi - \beta_2)]^{1/2} \\
\varphi^{(0)} = 8\pi M_0 \sin\theta^0 (z - d/2) \qquad 0 \leqslant z \leqslant d \\
\tilde{\varphi}^{(0)} = \begin{cases} 4\pi M_0 d \sin\theta^0 & z \geqslant d \\ -4\pi M_0 d \sin\theta^0 & z \leqslant 0. \end{cases}
\end{gathered} \tag{14}$$

The magnetostatic potential outside the slab is determined by solving the Dirichlet problem. In particular, we have in the $z \geqslant d$ range

$$\begin{gathered}
(\partial_\eta^2 + \partial_z^2)\tilde{\varphi}^{(1)} = 0 \qquad \tilde{\varphi}^{(1)} \to 0 \qquad (z \to \infty) \\
\tilde{\varphi}^{(1)}(\eta, \tau, z = d) = \varphi^{(1)}(\xi, \tau, z = d).
\end{gathered} \tag{15}$$

The boundary conditions express the continuity of the potential at the boundary between two media. The solution of the boundary problem (15) takes the form

$$\tilde{\varphi}_1^{(1)} = \frac{1}{\pi} \int_{-\infty}^{\infty} \frac{\tilde{\varphi}^{(1)}(\eta', z = d)(z - d)}{(z - d)^2 + (\eta' - \eta)^2} \, \mathrm{d}\eta' \qquad (z \geqslant d) \tag{16}$$

and as a result we have at $z = d$

$$\partial_z \tilde{\varphi}^{(1)}|_d = \hat{H} \partial_{\eta'} \tilde{\varphi}^{(1)}(\eta', z = d). \tag{17}$$

Here $\hat{H}$ is the Hilbert transformation:

$$\hat{H} u = \frac{1}{\pi} P \left(\int_{-\infty}^{\infty} \frac{\mathrm{d}x' u(x')}{x' - x} \right).$$

The symbol P denotes the principal-value integral. Equation (17) should be used for formulation of the boundary condition at $z = d$ in calculating the magnetostatic potential $\varphi^{(1)}$ inside the slab. Inasmuch as inside the slab the variable $\xi = \varepsilon\eta$ is employed, the limiting value of the derivative should be expressed in terms of ξ. From (15) and (17) the following expression for the limiting value of the derivative $\partial_z \tilde{\varphi}^{(1)}|_d$ in terms of $\varphi^{(1)}$, ξ is obtained:

$$\partial_z \tilde{\varphi}^{(1)}|_d = \varepsilon \hat{H} \partial_{\xi'} \varphi^{(1)}(\xi', z = d) = \mathrm{O}(\varepsilon). \tag{18}$$

Similarly, we have for $z = 0$

$$\partial_z \tilde{\varphi}^{(1)}|_0 = -\varepsilon \hat{H} \partial_{\xi'} \varphi^{(1)}(\xi', z = 0) = \partial_z \tilde{\varphi}^{(1)}|_d = O(\varepsilon). \tag{19}$$

Equations (18) and (19) show that the limiting values of the derivative $\partial_z \tilde{\varphi}^{(1)}|_{0,d}$ belong to the second-order term of reductive perturbation theory in ε. The magnetostatic potential within the slab is determined by a simpler way in the first order of the perturbation theory in ε:

$$\partial_z^2 \varphi^{(1)} = 0 \qquad (0 \leqslant z \leqslant d)$$
$$[-\partial_z \varphi^{(1)} + 8\pi M_0 \cos\theta^0 \, \theta_1^{(1)}(\xi, \tau)]|_{0,d} = 0.$$

The solution has the form $\varphi^{(1)} = 8\pi M_0 \cos\theta^0 \, \theta_1^{(1)}(\xi, \tau)(z - d/2)$. The analysis of the high-order equations is performed similarly. The calculation of the second-order magnetostatic potentials $\varphi^{(2)}$ inside the film reduces to the solution of the following boundary value problem:

$$\begin{aligned} &\partial_z^2 \varphi^{(2)} = 0 \qquad 0 \leqslant z \leqslant d \\ &-\partial_z \tilde{\varphi}^{(1)}|_{0,d} = [-\partial_z \varphi^{(2)} + 4\pi M_0 (\cos\theta^0 \, (\theta_1^{(2)} + \theta_2^{(2)}) - \sin\theta^0 \, \theta_1^{(1)2})]|_{0,d}. \end{aligned} \tag{20}$$

The solution of equation (20) is trivial because the functions $\theta^{(1)}$ and $\theta_2^{(2)} + \theta_1^{(2)}$ are z independent. As a result the second order and third order of the perturbation theory give the effective equation for the function $\theta_1^{(1)}(\xi, \tau)$:

$$\begin{aligned} &\partial_\tau \theta_1^{(1)} - \nu \hat{H} (\partial_{\xi'}^2 \theta_1^{(1)}) + q \partial_\xi (\theta_1^{(1)})^2 = 0 \\ &\nu = 2\pi d M_0 \gamma \cos\theta^0 \qquad q = \tfrac{3}{2} s \tan\theta^0 \end{aligned} \tag{21}$$

which coincides with the Benjamin–Ono equation. The Benjamin–Ono model admits the multi-soliton excitations [8, 9] and can be investigated in detail using the inverse scattering method [10, 11].

4. Analysis of soliton solutions

The magnetostatic soliton is less localized in comparison with the solitons of exchange and exchange–relativistic origin. On the contrary the one-soliton solution of equation (21) is described by the 'algebraic' wave rather than by the exponential wave:

$$\begin{aligned} &\theta_1^{(1)} = -\sigma \Delta / [(\xi + v\tau)^2 + \Delta^2] \\ &\Delta = (g M_0 2\pi d \gamma \cos\theta^0)/v > 0 \qquad \sigma = (4\pi d \cot\theta^0)/3[\delta + 4\pi - \beta_2] \end{aligned} \tag{22}$$

where v is the positive real parameter. In terms of the initial variables (x, t) the soliton (22) corresponds to the localized excitation propagating at the velocity $v + s$. Note that the soliton amplitude is proportional to the excess of its velocity above the phase velocity of the spin wave. The solitons arise probably in a threshold way as a result of the increase in the amplitude of exchange–magnetostatic spin waves and are realized only in dynamics. The system tends to reduce its energy by radiation of solitons. The size of the soliton must exceed the thickness of a slab (see (9)). This requirement bounds the interval of the values of the parameter v: $v/s \sim d(\Delta\delta)^{-1} \ll 1$.

Equation (21) describes the dynamics of the Lagrangian system. With the help of equations (14)–(20) it can be shown that, in the approximation considered, the total expression (6) for the Lagrange function density is reduced to the effective formula

$$L_{\text{eff}} = c[\tfrac{1}{2}\partial_\xi\chi\partial_\tau\chi - \tfrac{1}{2}\nu\partial_\xi\chi\partial_\xi\hat{H}\partial_{\xi'}\chi + \tfrac{1}{3}q(\partial_\xi\chi)^3]. \tag{23}$$

Here $c = 4M_0a^2(g\gamma)^{-1}\cos\theta^0$, a is lattice constant and we have introduced the potential $\partial_\xi\chi = \theta_1^{(1)}$ instead of the field $\theta_1^{(1)}$. The variation in the action corresponding to L_{eff} with respect to the field χ gives equation (21). Equation (23) allows one to obtain the expressions for the integrals of motion, namely for the energy E and field momentum P:

$$\begin{aligned} E &= \int \mathrm{d}\xi \left[\frac{\partial L_{\text{eff}}}{\partial(\partial_\tau\chi)}\partial_\tau\chi - L_{\text{eff}}\right] = c\int \mathrm{d}\xi \left(\tfrac{1}{2}\nu\theta_1^{(1)}\partial_\xi\hat{H}\theta_1^{(1)} - \tfrac{1}{3}q(\theta_1^{(1)})^3\right) \\ P &= \int \mathrm{d}\xi \frac{\partial L_{\text{eff}}}{\partial(\partial_\tau\chi)}\partial_\xi\chi = \frac{c}{2}\int \mathrm{d}\xi\, (\theta_1^{(1)})^2. \end{aligned} \tag{24}$$

Substituting the solution (22) into (24) we obtain

$$E = d\pi\sigma a\Delta^{-1}M_0\cos\theta^0 \qquad P = \pi(\sigma a)^2M_0\cos\theta^0(g\gamma\Delta)^{-1}. \tag{25}$$

Let us note that the relations obtained are related to the coordinate system moving with the velocity s. The dispersion law of the spin waves in this reference system has the form

$$\Omega(k) = \omega(k) - ks = 2\pi d\gamma M_0 g\cos\theta^0|k|k. \tag{26}$$

Equations (25) and (26) allow one to interpret the soliton (22) as a complex consisting of N non-interacting magnons with the wavevectors $k = (2\Delta)^{-1}$. Here 2Δ is the parameter characterizing the soliton width. The number N is defined by the formula

$$N = \pi(\sigma a)^2M_0\cos\theta^0(\gamma\mu_{\text{B}})^{-1} \sim \pi[8\pi d(3ah)^{-1}]^2 \gg 1 \tag{27}$$

where μ_{B} is the Bohr magneton. If we take into consideration that $g = 2\mu_{\text{B}}/\hbar$, equation (25) can be rewritten in the form

$$P = N\hbar k \qquad E = N\hbar\Omega(k) \qquad k = (2\Delta)^{-1} \tag{28}$$

where $\Omega(k)$ coincides with the dispersion law of the spin wave (26), $k > 0$. The soliton energy E can be written in the following form:

$$E = P^2/2M_{\text{eff}}. \tag{29}$$

This allows one to interpret the soliton as an object similar to the particle with effective mass

$$M_{\text{eff}} = Nm_0 \tag{30}$$

where $m_0 = \hbar(4\pi d\gamma M_0\cos\theta^0)^{-1}$ is the effective mass of a single magnon with the dispersion law (26). The result obtained is typical for the 'algebraic' soliton since its energy, momentum and mass are the sums of N corresponding quantities for a single magnon. However, in the present problem, N is not a free physical parameter, as the velocity v or momentum P is. It is defined by external conditions, namely the slab thickness d and the magnitude of the external magnetic field h. From this point of view the soliton energy depends only on its momentum P and is, by a factor of N, smaller than the magnon energy with the same momentum. We believe that the integrability of the model (21) assures the stability of the soliton (22). The multi-soliton solutions of (21) describe the elastic collisions between these 'algebraic' solitons.

The examination of the role of dissipation is an important problem in soliton physics. The calculation of the relaxation rate for the spin waves due to the exchange interaction leads to the following expression: $\Gamma = \mu k^2$ (μ = constant > 0) [12]. In the case under consideration the relaxation can be taken into account in a phenomenological manner by adding the term $-\mu\partial_\xi^2\theta_1^{(1)}$ on the left-hand side of equation (21):

$$\partial_\tau\theta_1^{(1)} - \nu\hat{H}(\partial_{\xi'}^2, \theta_1^{(1)}) - \mu\partial_\xi^2\theta_1^{(1)} + q\partial_\xi(\theta_1^{(1)})^2 = 0. \tag{21a}$$

It is remarkable that the obtained equation (21a) admits a wide class of exact solutions [13, 14]. The simplest of these has the form

$$\begin{aligned} &\theta_1^{(1)} = -\sigma\{[\xi + \xi_0(\tau)]^2 + \Delta^2(\tau)\}^{-1}\{\Delta(\tau) + \mu/\nu[\xi + \xi_0(\tau)]\} \\ &\xi_0(\tau) = (\nu/\mu)[\Delta(\tau) - \delta_0] \qquad \Delta(\tau) = \{2\mu\tau + \delta_0^2\}^{1/2} \qquad \delta_0 > 0. \end{aligned} \tag{22a}$$

Here δ_0 is the problem parameter. In the $\mu \to 0$ limit this solution reduces to (22). From equation (22a) it follows that the localized excitation becomes blurred in time and extends in width, while the amplitude and velocity are diminished.

5. Results and discussion

It is known that the magnetic state of antiferromagnets is described by a two-vector order parameter, namely the ferromagnetism vector $\boldsymbol{M} = \boldsymbol{M}_1 + \boldsymbol{M}_2$ and antiferromagnetism vector $\boldsymbol{L} = \boldsymbol{M}_1 - \boldsymbol{M}_2$. In terms of $\theta_1^{(1)}$ and $\varphi_1^{(0)}$ they take the form

$$\begin{aligned} &\boldsymbol{M} = 2M_0(0; 0; \sin\theta^0 + \cos\theta^0\theta_1^{(1)} + O(\varepsilon^2)) \\ &\boldsymbol{L} = 2M_0(\cos\theta^0\cos\varphi_1^{(0)}; \cos\theta^0\sin\varphi_1^{(0)}; 0) + O(\varepsilon). \end{aligned} \tag{31}$$

From (31) it follows that the dynamic changes in $\boldsymbol{L}$ take place in zero order in ε and the dynamic changes in $\boldsymbol{M}$ appear in the first order of magnitude in ε. In the localization region of a soliton the component M_3 is smaller than at infinity; so M_3 describes a 'dark' soliton. The depth of this minimum is proportional to the parameter v. In correspondence to (14), $\varphi_1^{(0)}$ is defined by the expression

$$\varphi_1^{(0)} = -(\sigma/\gamma)\tan^{-1}[(\xi + v\tau)/\Delta]. \tag{32}$$

According to (31) and (32) the vector $\boldsymbol{L}$ lies in the plane of the slab being turned by an angle $\Delta\varphi_1^{(0)}$ in the region of localization of a soliton:

$$\Delta\varphi_1^{(0)} = (\pi\sigma/\gamma) \sim (8\pi^2 d/3ah) \gg \pi$$

i.e. completing many revolutions.

It can be shown that in weak magnetic fields ($h/2\delta < \theta_1^{(1)}$) the effective equations differ from (21) and have no solutions of the 'algebraic' soliton type. In this interval of the parameter values the interaction of Goldstone modes is less intensive and defined by the cubic (rather than quadratic as in (21)) term in the field $\theta_1^{(0)}$ in an effective equation. It is of interest that in the infinite antiferromagnet the dynamics of non-linear small-amplitude waves are described by local equations of the Korteveg–de Vries type. These equations have different forms depending on the magnetic field magnitude and wavevector direction. Certainly in the case of a thick film, 'exponential' solitons are realized rather than 'algebraic' solitons. The reason is that in the infinite sample the magnetostatic interaction contribution is of secondary importance in comparison with the short-range exchange interaction contribution. In contrast, in a thin film the long-range dipole–dipole forces dominate and the surface magnetic charges form in principle the 'algebraic' solitons. Other magnetic charges, e.g. volume charges, refine only the internal structure of solitary waves. The model considered above is sufficiently simple and consistent with real antiferromagnets. The specific evaluations can be performed using the material parameters for $MnCO_3$ (Néel point $T_N = 325$ K), α-Fe_2O_3 with $T_N = 950$ K having an 'easy-plane' anisotropy above the Morin point $T_M = 260$ K, and $FeBO_3$ with $T_N = 348$ K. In these systems it is easy to realize the conditions (9) and (10) when $d \simeq 5$–10 μm. Let us evaluate the parameter v for $FeBO_3$. Considering that for this material $\delta \simeq 10^3$ and $M_0 = 1040$ Oe, we obtain $v \leqslant 10^2$ cm s^{-1} under conditions (10). The effective soliton velocity $v+s$ is thereby proved to be sufficiently high, i.e. of the order of 10^4–10^5 m s^{-1} and is close to the value of the velocity of sound propagation in these materials. Evidently, the presence of magnetostatic solitons can be revealed through their resonance interaction with the elastic subsystem of the antiferromagnet. We believe that the approach outlined allows one to approximate correctly the weakly non-linear dynamics of quasi-one-dimensional waves in magnetic films.

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